



# How to plan lessons effectively using the 5 big ideas

Boolean Conference 2019

Ella Keddie

[ella.keddie@colstonsgirls.org](mailto:ella.keddie@colstonsgirls.org)

MathsHUBS

What do you see?

$96 - 5x$   
 $36 + 5(12 - x)$   
 $8(12 - x) + 3x$

National Centre for Excellence in Mathematics

# How do you currently do it?

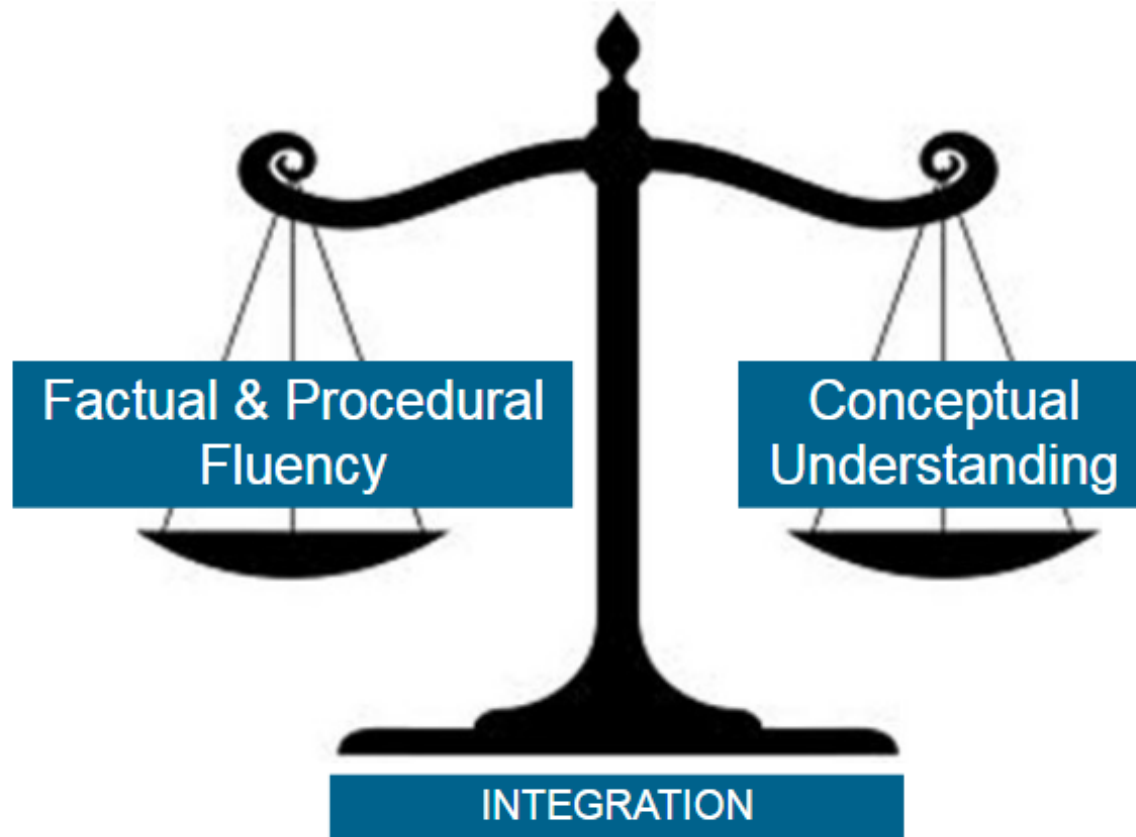


Developing and  
planning a mastery  
curriculum that is  
effective.



# Teaching for Mastery is...

## The New Curriculum



# How we can plan effectively:

**S-Planning:** Incorporating well structured and thought through learning objectives and outcomes.

## Features of Mastery

|   |   |
|---|---|
| <i>Curriculum design</i>                | Longer units of work, prioritising key topics   |
| <i>Lesson design</i>                    | Carefully structured lesson to develop the detail and depth   |
| <i>Resources</i>                        | Quick intervention  |
| <i>Teaching methods differentiation</i> | Carefully chosen examples and activities. Application of variation theory. Effective use of representations |
| <i>Productivity and practice</i>        | Keeping the class together and aiming for depth   |
|   | Intelligent practice  |

Looking at the 5 Big Ideas to generate depth of learning within the classroom.

# Curriculum Design (S-Planning)

## Shallow Learning

- Surface learning,
- Temporary, often lost

## Meeting Expectations

- With support being able to meet the objectives outlined in the National Curriculum

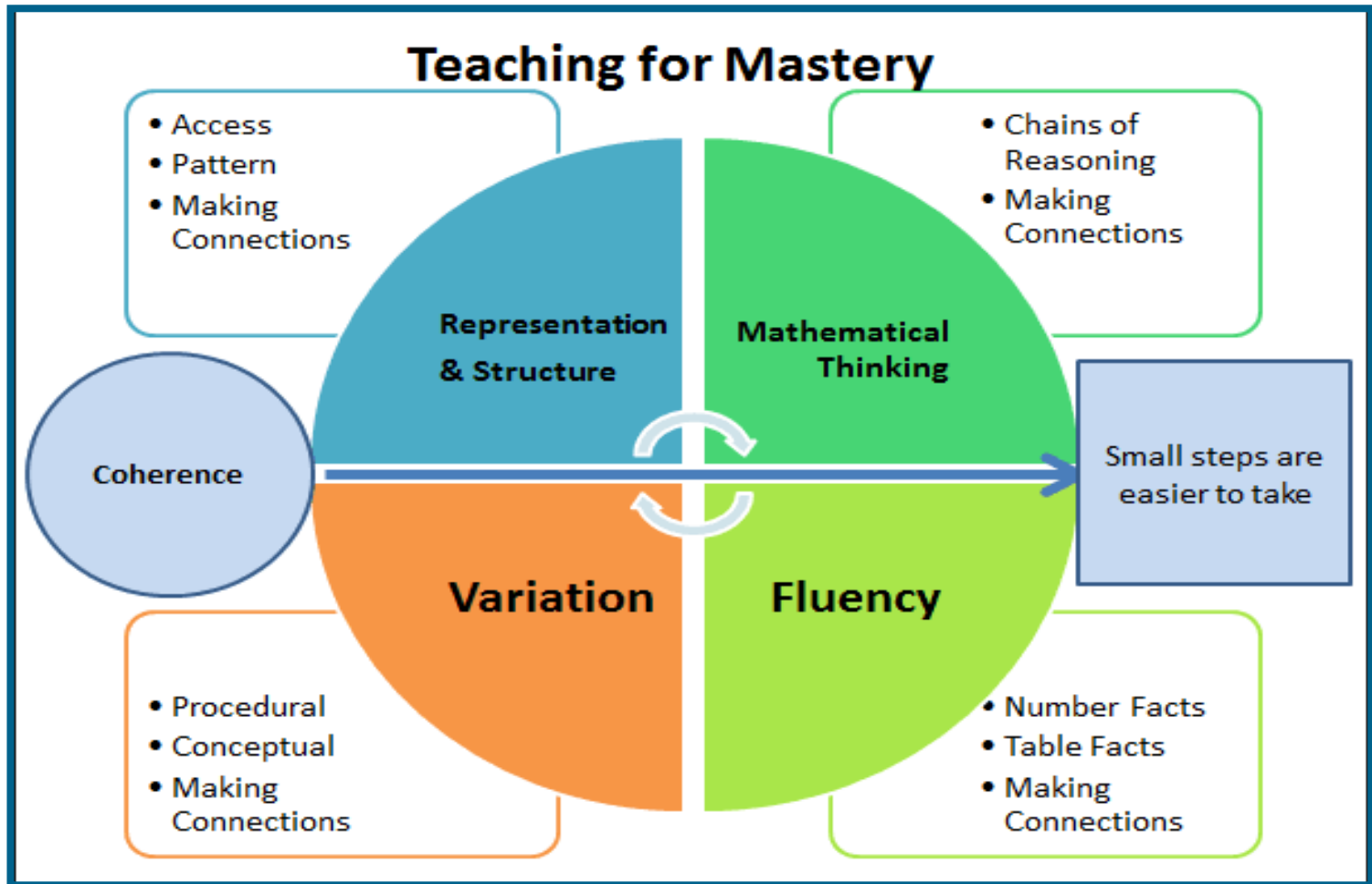
## Mastery

- Obtaining greater level of understanding and being able to apply learning in different context

## Working at Greater Depth

Learning be transferred and applied in different contexts  
Pupils can explain their understanding to others

# The 5 big ideas



# What areas are you prioritising in schools?

Year 7 Only

Number and Place Value  
Addition and Subtraction  
Multiplication and Division  
Fractions and ratio

Measurement

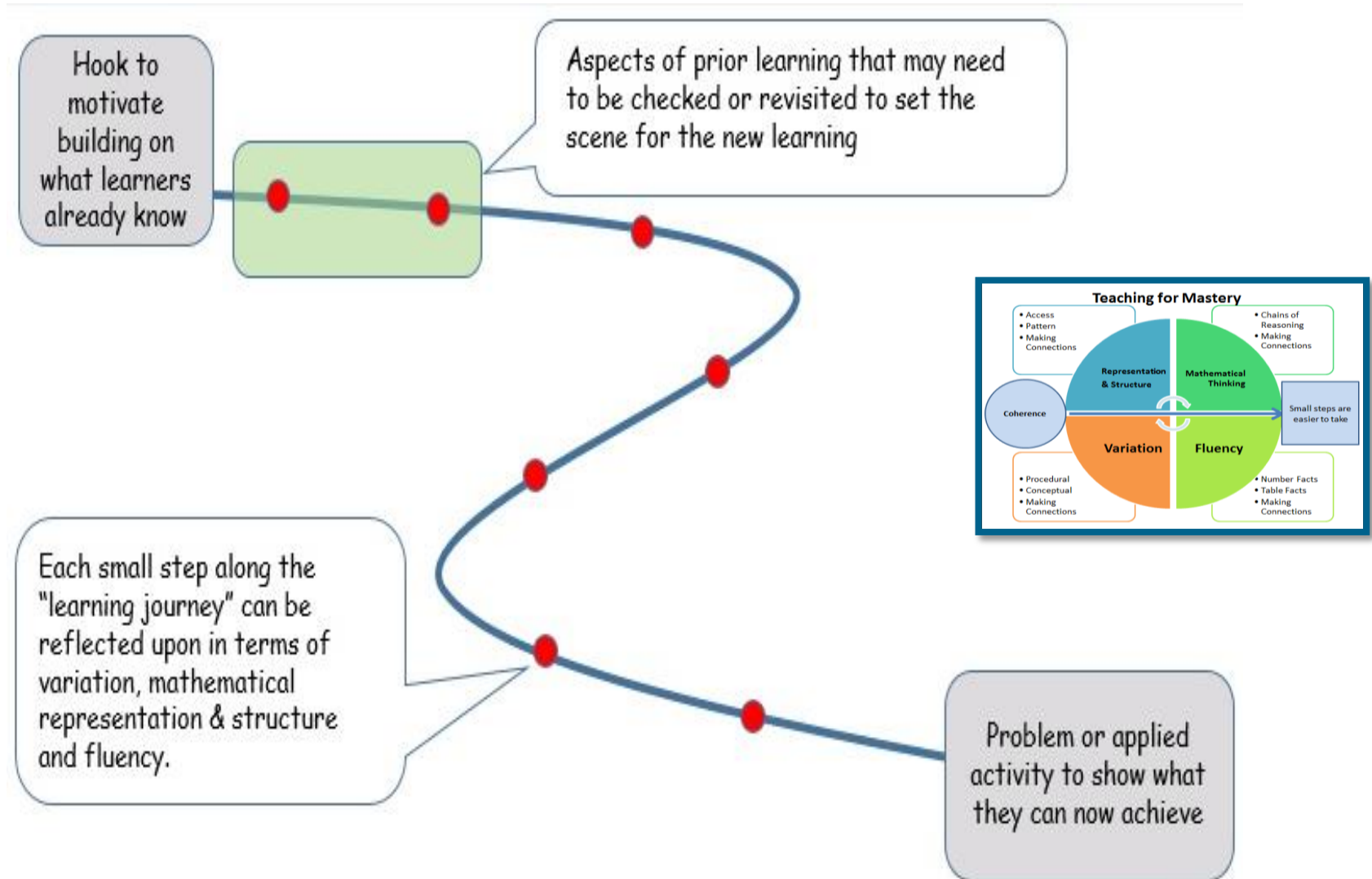
Geometry

Statistics

Algebra

**What about you and your schools?**

# S-plan: Identifying the connections between lessons in a topic area.





# Your S-Plan: Learning Journey

## (Topic Plan )



### Planning

- Spend longer time on topics
- Prioritise key topics
- Focus on relationships
- Make Connections
- Ensure intelligent practice

### The Key:



What's your hook?



What are the individual small steps you need the students to achieve? (label on diagram)



Prior Learning Needed? When will you cover this on the plan?

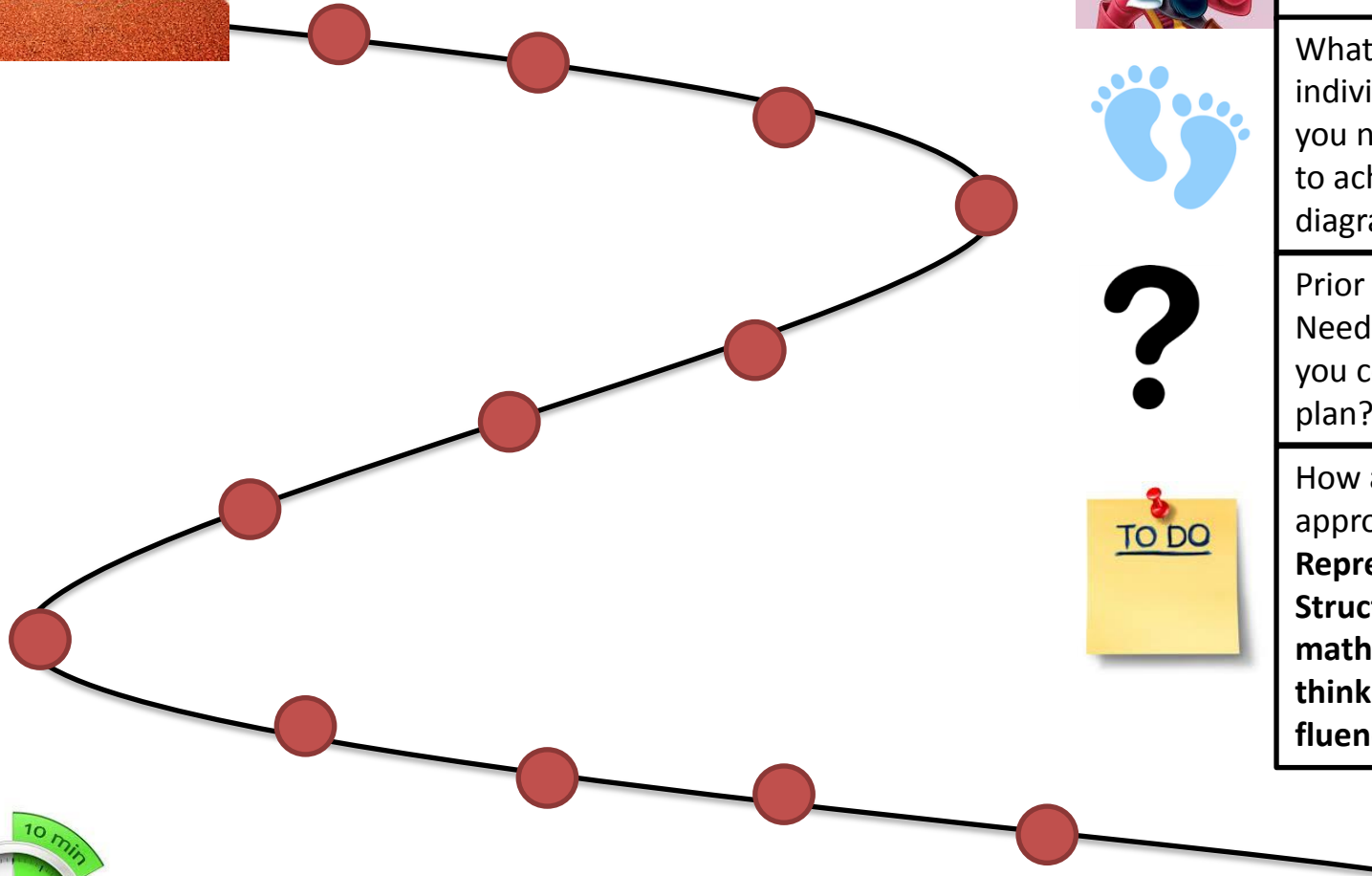


How are you going to approach these task?  
**Representation & Structure, variation, mathematical thinking, fluency**



# Your S-Plan: Learning Journey

## (Topic Plan )



### The Key:



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Lesson Planning & Resources  
available

# Big Idea: Variation

## Key Messages:

1. The central idea of teaching with variation is to highlight the essential features of a concept or idea through varying the non-essential features.
2. Variation is not the same as variety – careful attention needs to be paid to what aspects are being varied (and what is not being varied) and for what purpose.
3. When giving examples of a mathematical concept, it is useful to add variation to emphasise:
  - a. What it is (both standard and non-standard examples);
  - b. What it is not.
4. When constructing a set of activities or questions it is important to consider what connects the examples; what mathematical structures are being highlighted? Students are encouraged to avoid mechanical practice and, instead, to practice the thinking process (intelligent practice)

## Variation

- Procedural
- Conceptual
- Making Connections

# Variation in Greater Depth



## Purpose of Variation

Develop deep learning rather than superficial learning

Provide the necessary repetition to embed and sustain learning

Make connections between concepts

A yellow sticky note with a red pushpin at the top left corner. The word "Recap" is written on the note in a black, cursive font.

Recap

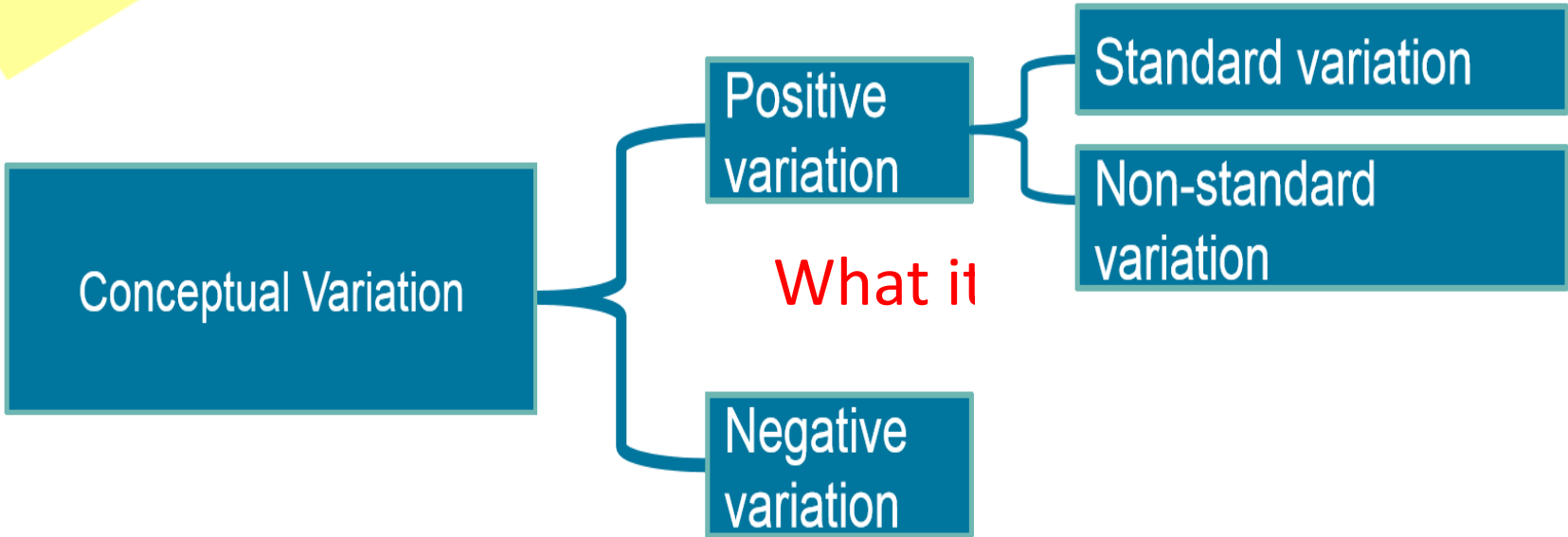
# Conceptual Variation:



Recap

Drawing attention to what is to be learnt – the object of the learning, the essence of the concept.

Leading to generalisation



What it

What it is not

# Procedural Variation



Provides the opportunity

- for practice (intelligent rather than mechanical);
- to focus on relationships, not just the procedure;
- to make connections between problems;
- to use one problem to work out the next;
- to create other examples of their own.



# Variation to create depth: it's the subtle changes....



|   |  |   |   |   |     |
|---|--|---|---|---|-----|
| $312 \times 3 =$                                      |  |   |   |   |     |
|   |  | H | T |   |     |
| $300 \times 3 =$                                      |  |   |   |   |     |
| $10 \times 3 =$                                       |  |   |   |   |     |
| $2 \times 3 =$  |  |   |   |   |     |
| $132 \times 3 =$                                      |  |   |   |   |     |
|   |  |   |   |   |     |
| $21 \times 3 =$                                       |  | H | T | O |     |
| $\begin{array}{r} \times 20 \quad 1 \\ 3 \end{array}$ |  |   | 2 | 1 |     |
|   |  |   |   |   | 3 x |

$231 \times 3 =$

|  |   |   |   |
|--|---|---|---|
|  | H | T | O |
|  | 2 | 3 | 1 |

$5 \times 30 = \square$   
 $5 \times 30 = 5 \times 3 \times 10 = \square \times 10 = \square$

$5 \times 3 = 15$   
 $5 \times 30 = 150$

When the number in the multiplier becomes 10 times as much, the answer also becomes 10 times as much.

$444 \times 4 =$

$225 \times 4 =$

What do you notice about the answers to these questions?

- a)  $43 \times 7 =$
- b)  $421 \times 6 =$
- c)  $5 \times 731 =$
- d)  $845 \times 6 =$
- e)  $\square = 4 \times 368$

Write a mathematical sentence for  $97 + 65 =$

$85,021 + 45,10 =$

One hundred thousand add fifty seven = ?

Emma has 5 different British coins. What different combinations could she have?

One pound = ? add 50p. Find the possibilities

$-1 + 4 =$

$674 + 247 =$

$47 = ? + ?$  Find the possibilities

Find the quickest way to add the numbers 1-20.

**ADDITION**

Find all the possibilities:

one side of a square measures 4.5mm. What is the perimeter?

How many different names are there for addition?

$3:20\text{pm} \text{ minus } 9 \text{ hours } 57 \text{ minutes} =$

How many ways can you make 35 using just addition?

$57 + 47 =$

$3y + 2x =$

$? + 15 = 21$

# Greater Depth through Variation:

- Example 1: Fluency

*Fluency:*

|    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| x  | 2 | 4 | 8 | 3 | 6 | 9 |
| 5  |   |   |   |   |   |   |
| 7  |   |   |   |   |   |   |
| 11 |   |   |   |   |   |   |
| 12 |   |   |   |   |   |   |

# Greater Depth through Variation:

- Example 1: Reasoning

|   |    |    |    |    |   |    |
|---|----|----|----|----|---|----|
| x | 2  |    |    | 5  | 6 |    |
|   | 20 |    | 40 | 50 |   |    |
| 9 |    | 27 |    |    |   |    |
| 8 |    |    |    |    |   |    |
|   | 14 |    |    |    |   | 49 |

# Greater Depth through Variation:

- Example 1: Problem Solving

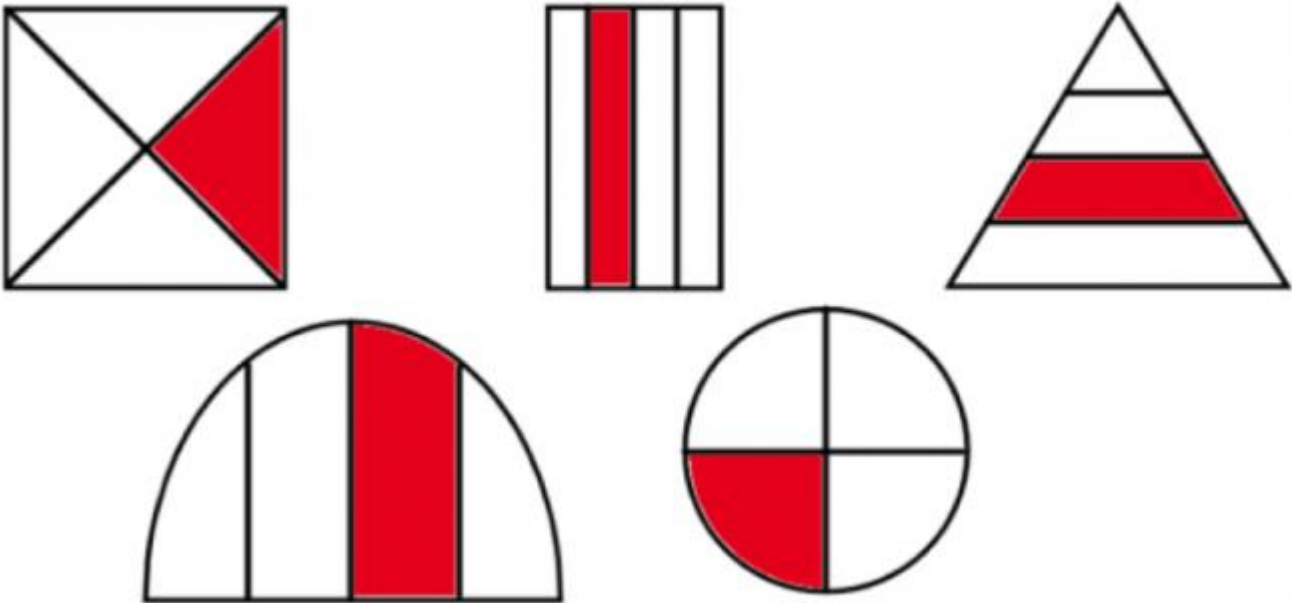
Using the digits 1 – 9 only once, find the missing factors in this grid.

|   |    |   |  |    |  |
|---|----|---|--|----|--|
| X |    |   |  |    |  |
|   | 16 |   |  |    |  |
|   |    | 7 |  |    |  |
|   |    |   |  |    |  |
|   |    |   |  | 18 |  |

# Greater Depth through Variation:

- Example 2: Fluency

Tick which shapes have a quarter shaded...



# Greater Depth through Variation:

- Example 2: Reasoning

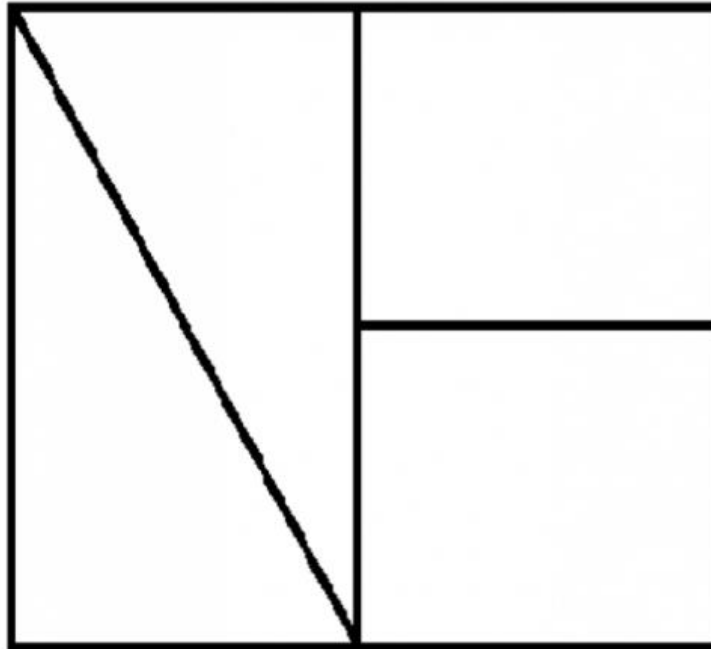
How many different ways can you divide a square into quarters?



# Greater Depth through Variation:

- Example 2: Problem Solving

Dani says this square is split into quarters. Ryan says that can't be correct. Who do you agree with?



# Example 1: Discuss

What are these examples  
examples of ?

**Q1** Solve the following simultaneous equations:

**a)**  $4x + 6y = 16$   
 $x + 2y = 5$

**b)**  $3y - 8x = 24$   
 $3y + 2x = 9$

**c)**  $3y - 10x - 17 = 0$   
 $\frac{1}{3}y + 2x - 5 = 0$

**d)**  $\frac{x}{2} - 2y = 5$   
 $12y + x - 2 = 0$

**e)**  $3x - 4y = 5x - 14$   
 $2y + x = 11y - 26$

**f)**  $3x + 4y = 10$   
 $5x - 7y = 3$

**g)**  $2y - 3x = 1$   
 $4x + 5y = 37$

**h)**  $10x - 7y = -9$   
 $8x + 9y = 22$

**i)**  $\frac{3}{5}x + 2y = 21$   
 $2x - \frac{2}{3}y = 4$



## Example 2: Discuss

### Standard or Non-standard

$$3x + 1 = x + 5$$

$$3x + 2 = x + 6$$

$$4x + 1 = 2x + 5$$

$$6x + 2 = 2x + 10$$

$$3x + 1 = x$$

$$x + 2 = 3x + 6$$

$$2x + 1 = 4x + 5$$

$$6x = 2x + 10$$

## Example 3: Discuss

True or False

$$\frac{4}{9} = \frac{5}{10}$$

$$\frac{6}{8} = \frac{3}{16}$$

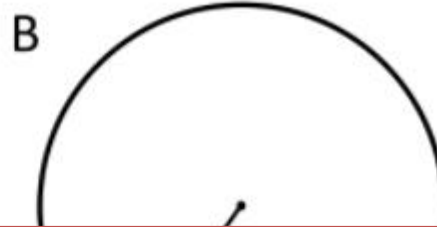
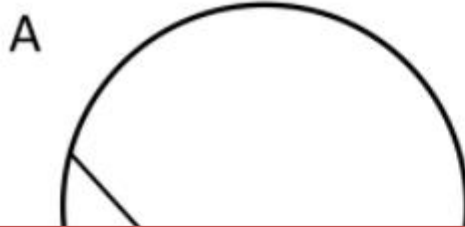
$$\frac{3}{5} = \frac{3}{10}$$

$$\frac{3}{7} = \frac{6}{14}$$

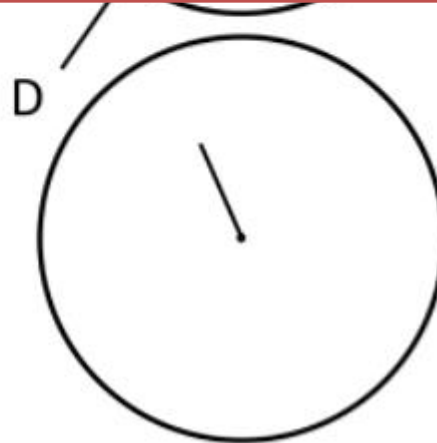
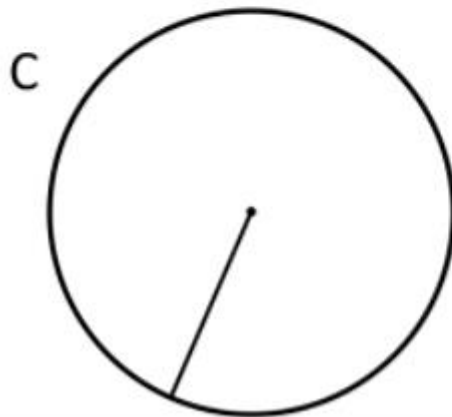
**Conceptual: Negative variation**

# Example 4: Discuss

**Concept / non concept** radius of a circle



**Conceptual: Negative variation**



# Now it's your turn!

- On your tables there is a planning sheet incorporating variation theory into your lessons.
- You need to **create** an activity that will support.
- **Think about:** How can you adapt it?

## CREATE AN INEQUALITY, GIVEN A SOLUTION SET

Directions: Write an inequality whose solution set is  $x < -1/2$  by filling in the boxes with whole numbers 0 through 9, using each number at most once.

$$\square x + \square > \square x + \square$$

## CREATE AN EQUATION WITH A SOLUTION CLOSEST TO ZERO

Directions: Using whole numbers 1 through 9 at most once, create an equation such that the solution is closest to zero.

$$\square x + \square = \square x + \square$$

## THE TRIANGLE INEQUALITY

Directions: Use the digits 1 through 10 (without repeating any number) to complete the scenarios below:

$$9, \square, \square$$

The triangles represented by these sides can not exist.

$$\square, \square, \square$$

The triangles represented by these sides is right.

$$\square, 7, \square$$

The triangles represented by these sides can not exist.

$$\square, \square, \square$$

The triangles represented by these sides is right.

## PARTITIONING A LINE SEGMENT

Directions: Fill in the boxes by using the numbers 1-8 exactly once. Create a line segment AB, where between point A and point B, there exists a point P so that it partitions line segment AB into a ratio.

$$A: (\square, \square)$$

$$B: (\square, \square)$$

$$P: (\square, \square)$$

$$\text{Ratio } \square : \square$$

[www.openmiddle.com](http://www.openmiddle.com)

$$\square x + \square = \square x + \square$$

## Variance Planning Sheet

Minimally different examples  
Standard vs Non Standard

Concept vs Non Concept  
True or False

Key Concept

# Useful sites:

- Increasingly Difficult Questions
- Open Middle
- MathsPad (subscription approx. £50?)
- Jo Morgan (Resourceaholic): Topics in Depth project
- Mathsbot
- Naveen Rizvi blog
- Kris Boulton blog

# Additional Online Resources to help to support:

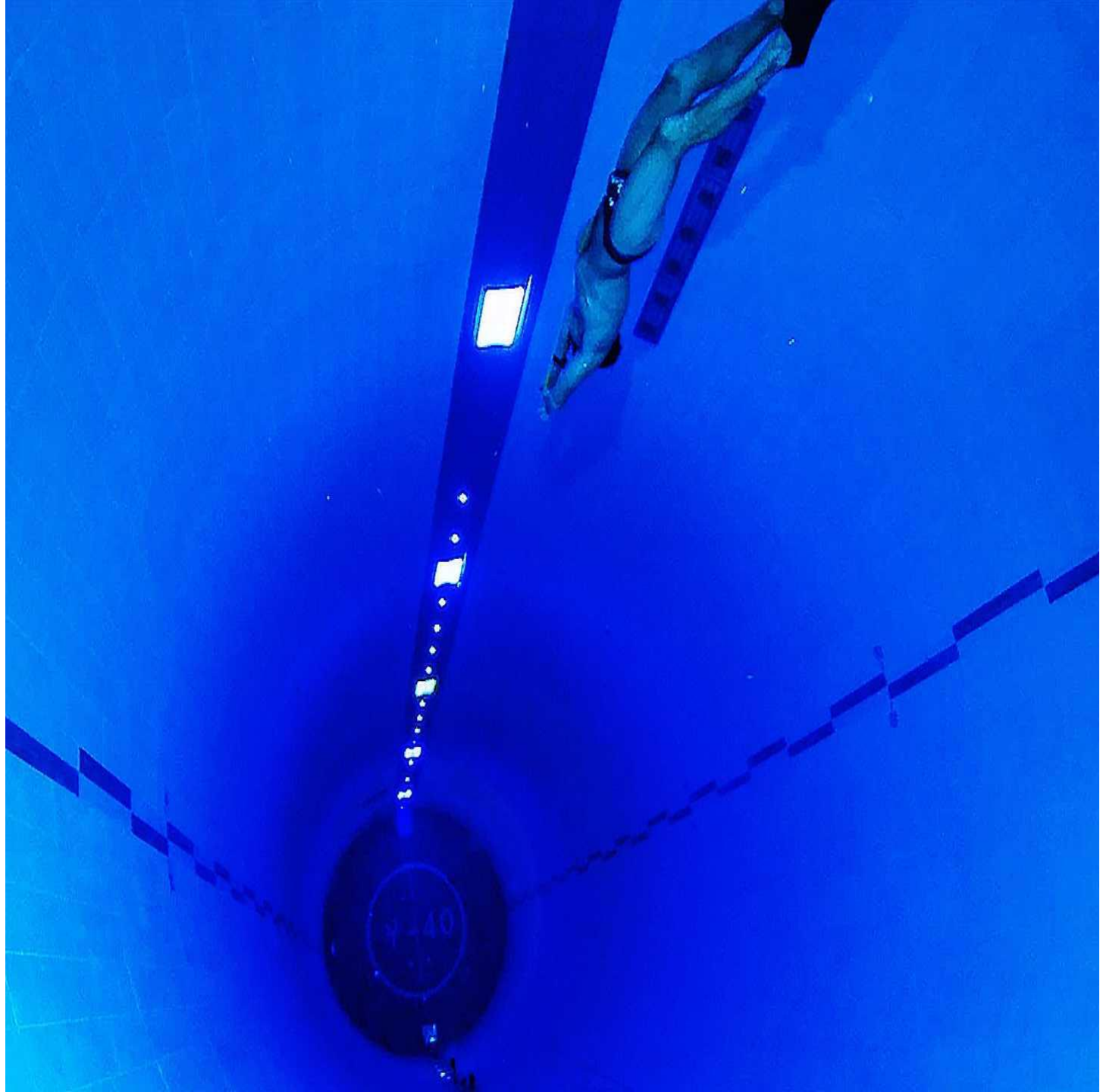
- Increasingly difficult questions: <http://taylorda01.weebly.com/increasingly-difficult-questions.html>
- Don Stewart: <http://donsteward.blogspot.co.uk/>
- The Maths Shed: <http://www.mathematicshed.com/the-singapore-maths-shed.html>
- NCETM- Secondary: [https://www.ncetm.org.uk/files/66633120/secondary\\_assessment\\_materials\\_november\\_2017.pdf](https://www.ncetm.org.uk/files/66633120/secondary_assessment_materials_november_2017.pdf)
- Jurassic Maths Hub work: <http://www.jurassicmaths.com/resources/mastery/variation/>
- Gareth Metcalfe ( primary school blogger): <https://garethmetcalfe.wordpress.com/2014/12/28/mastery-in-mathematics-deep-learning-for-the-able-mathematician/>
- Same but different questions: <https://ssddproblems.com/>
- Variation theory. Com <https://variationtheory.com/>
- <https://nonexamples.com/>

# Creating Activities / Questions that Promote Depth

Alternative Options/ Ideas







Change into  
improper fractions:

$$\begin{array}{l} 1 \\ ) \end{array} 1 \frac{3}{4} \quad \begin{array}{l} 2 \\ ) \end{array} 2 \frac{3}{4} \quad \begin{array}{l} 3 \\ ) \end{array} 3 \frac{3}{4}$$

$$\begin{array}{l} 4 \\ ) \end{array} 5 \frac{1}{3} \quad \begin{array}{l} 5 \\ ) \end{array} 5 \frac{2}{7} \quad \begin{array}{l} 6 \\ ) \end{array} 2 \frac{1}{3}$$

Challenge:

$$28 \frac{3}{17} = \frac{479}{17}$$

Using this information,  
write the following as  
improper fractions:

a)  $26 \frac{3}{17}$

b)  $30 \frac{2}{17}$

# Can you match up the calculations with each of the descriptions?

$$\pi \times 3.5^2$$

Finds the area of a circle with radius 7cm

Finds the area of a circle that fits perfectly inside a 4cm square

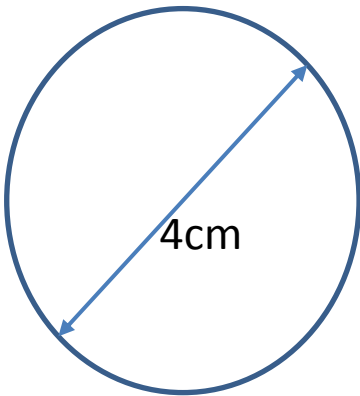
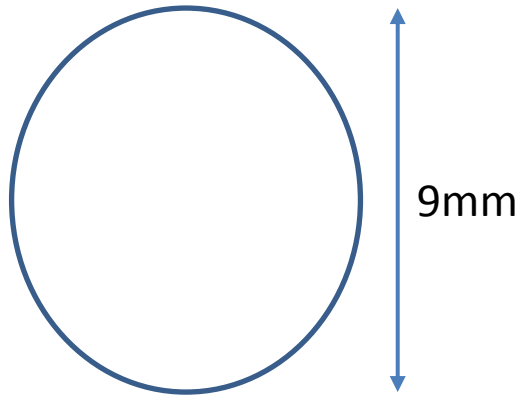
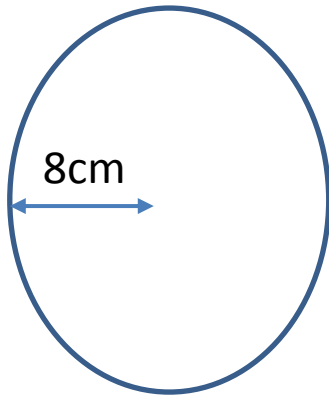
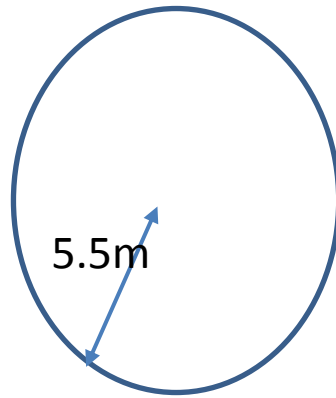
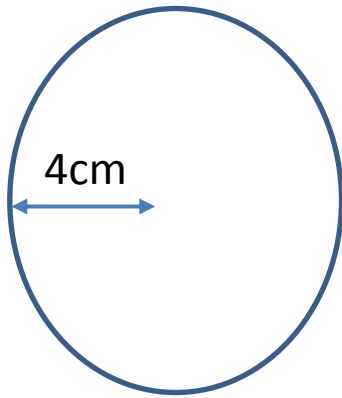
Finds the area of a circle with diameter 7cm

$$\pi \times 4^2$$

$$\pi \times 7^2$$

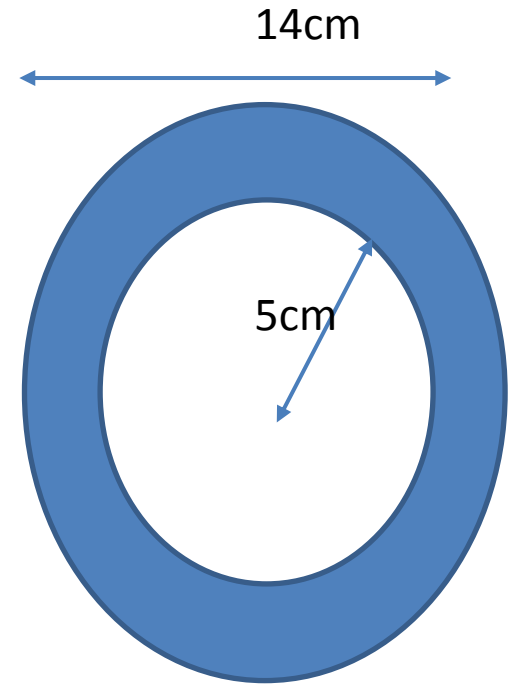
Finds the area of a circle with radius 4cm

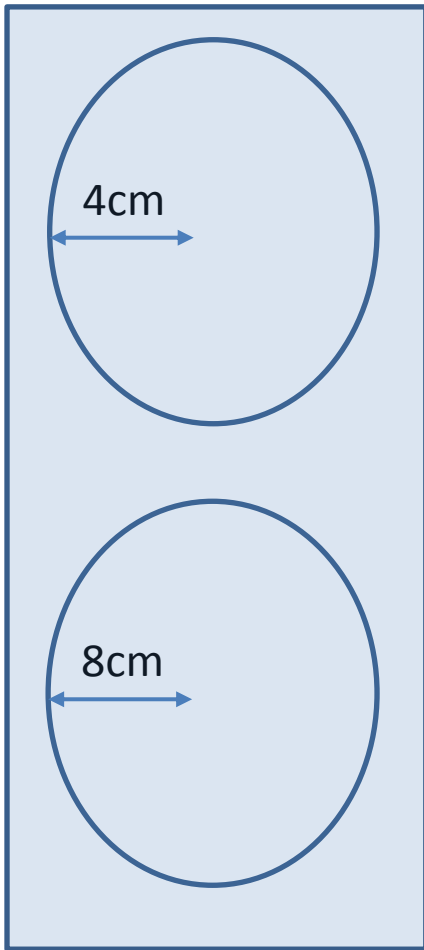
There should be one description left over. Can you write a calculation for it?



The diagram below is made from two concentric circles (circles with the same centre).

Calculate the Area of the blue section of the diagram:





50.3

201

Dave is unhappy with the answers to Q1 and Q2.

He says:

“Because the radius on Q2 is twice as big as the radius on Q1, our answer for Q2 should be double Q1.”

“I can see that the second answer is not double the first answer, so

something must have gone wrong.”

original radius

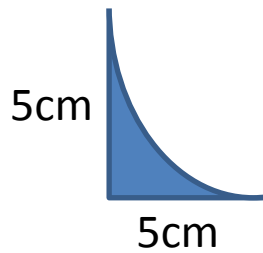
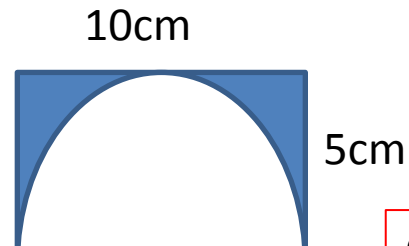
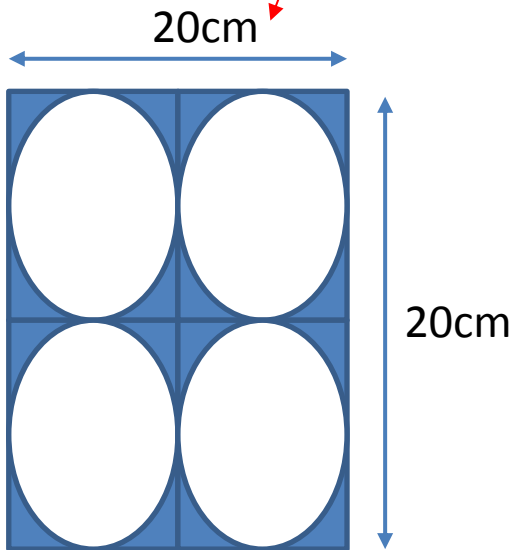
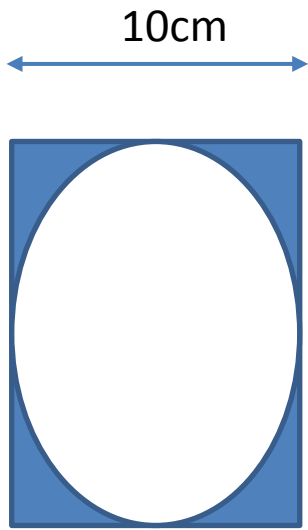


doubled radius



Because we are squaring, if lengths double, Areas will x4 (2 squared)

The circle fits perfectly inside the square.  
Can you find the shaded area?



Once you have finished the initial question, can you solve the other three questions by doing one simple extra calculation each time?

Potential Task: Solve the following quadratic equations:

$$y^2 - 14 = 5y$$

$$y(y - 5) = 14$$

$$y^2 + y = 2(3y + 7)$$

$$2y^2 - 10y - 28 = 0$$

**Probing Questions:**

What are the differences?

What are the similarities?

What do you notice?



# Other Examples to demonstrate depth of understanding:



- True/ False
- Fill in the missing blank
- Correct my mistakes
- Prove that.... Why?
- Represent in a different way...

|                        |
|------------------------|
| 1, 2, 3, 4, 6, 7, 9, 0 |
| — — + 8 = — —          |
| — — X 5 = — —          |

What's my mistake?

Solve:  $42 \times 5$

$42 = 40 + 2$

So...

|   |   |
|---|---|
| $\begin{array}{r} 40 \\ \times 5 \\ \hline 200 \end{array}$ | $\begin{array}{r} 2 \\ \times 5 \\ \hline 12 \end{array}$ |
|---|---|

My answer:  $42 \times 5 = 212$

|                  |                      |                                |
|------------------|----------------------|--------------------------------|
| Represent it...  | $1 \div \frac{2}{3}$ | Represent it in another way... |
| Write a story... |                      | Explain...                     |

# To summarise:

## Features of Mastery

|   |   |
|---|---|
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| <i>Productivity and practice</i>        | Intelligent practice  |



Thank  
you!!